Department of Mathematics, Pattamundai College, Pattamundai

Numerical Methods and Scientific Computing
Core -8
Section- A

1. State the iterative formula for regula falsi method to solve $f(x)=0$.

2. Give an example of(a) algebraic (b) transcendental equation .

3. What are the two **types** of errors involving in the numerical computation?

4. What is the criterion for the convergence in Newton Raphson method?

5. Derive Newton's algorithm for finding the pth root of a number N.

6. Write the iterative formula of Newton Raphson method.

7. Show that the iterative formula for finding the reciprocal of N is
   $x_{n+1}=x_n[2-Nx_n.]$

8. For solving a linear system, compare Gaussian elimination method and Gauss - Jordon method.

9 State the principle used in Gauss- Jordon method.

10. Solve the following system of equations by Gauss- Jordon method
    $5x+4y=15, 3x+7y=12$

11. Write a sufficient condition for Gauss- Seidel method to converge.

12. Give two indirect methods to solve a system of linear equations.

13. Find the dominant eigen value of A= $\begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix}$ by Power method.

14. Define Eigen value and Eigen vector.

15. By Gauss Elimination method solve $x + y =2$ and $2x + 3y =5$.

16. Find the approximate real root of $xe^x-3=0$ in $1<x<1.1$ by method of False position.

17. Find inverse of A = $\begin{matrix} 1 & 3 \\ 2 & 7 \end{matrix}$ by Gauss-Jordon method.

18. When shall we not use Newton-Raphson method?

19. Determine the largest eigen value and corresponding eigen vector of the matrix $\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}$

20. Write down the condition for convergence in iteration method.

21. State Newton's Divide difference interpolation formula for unequal intervals.

22. Supply the conditions for a spline to cubic.

23. State Lagrange's interpolation formula.

24. What is the Lagrange's formula to find y, if three sets of values (x_0, y_0) ,$(x_1; y_1)$ and (x_2, y_2) are given.

25. What is the assumption we make when Lagrange's formula is used?

26. Use Lagrange's formula, to find the quadratic polynomial that takes these values x : 0 1   3   Then find y(2).

   y: 0 1   0

27. Define Divided difference.

28. Derive Newton's backward difference formula by using operator method.

29. State Gregory-Newton forward difference interpolation formula.

30. When Newton's backward interpolation formula is used.

31. Write down Newton's backward difference formula.

32. Obtain a divided difference table for the following data:

   x: 5   7   11   13   17
   y: 150 392. 1452 2366 5202

33. Give the inverse of Lagrange's interpolation formula.

34. State the properties of cubic spline.

35. Form the divided difference table for :

   x : -1   1   2   4
   y : -1   5   23 119

36. If $f(x) = 1/x^2$, find f(a, b) and f(a, b, c) by using divided difference.

37. Find the second divided difference with arguments a,b, c, If $(x) = 1/x$

38. What is the nth divided difference of a polynomial of the nth Degree?

39. Using Newton's divided difference formula find the missing value from the table .

x :   1   2   4   5   6

y   14   15   5   -   9

40. State any two properties of divided difference.

41. Using Newton's divided difference formula

determine f(3) from the data :

x: 0   1   2   4   5

y: 1   14   15   5   6

42. When does Simpon's rule give exact result?

.   43. What is the condition for Simpon's

3/8 rule and state the formula?

44. W hat are the errors involved in

Simpon's 1/3 and 3/8 rule's for the

numerical integration?

45. What are the error's in Trapezoidal

and Simpon's rule of numerical

integration?

Section –B

1   By differentiating Newton's backward difference formula, find the first

derivatives of the function (x).

2 . Write the principle for derivative using divided differences.

3   When can numerical differentiation be used ?.

4   State the formula of Simpson's 3/8 th rule.

5. Compare Trapezoidal rule and Simpson's 1/3 rule for evaluating numerical

integration.

6. Write down the formula to calculate errors in quadrature formulae

7.   By Taylor series method, find y(0.1)given y'=x+y, y(1)=0

8. Name the method which is Taylor's method of first order.

9. Why is meant by initial value problem and give an example for it.

10. In the deviation of Runge-Kutta formula, Why it is called fourth order?

11. State the special advantage of Runge-Kutta method over Taylor series method?

12. Write down Euler algorithm to the differential equation.

13. Why is the condition to apply Adam-Bash forth method?

14. Name   two self starting methods for solving ordinary differential equation

15. Use Newton's method to find the real root of

3x- cosx -l=0.

16. Apply Gauss Jordon method to solve the equations

x+y+z=9, 2x-3y+4z=13,3x+4y+5z=40.

17. So1ve by Jacobi iteration method correct to two

decimal places l0x+y-z=11., x+l0y+z=28.,

x+y+10z=35.61.

18. Obtain by power method the numerically largest eigen

value of the matrix

$$\begin{matrix} 15 & -4 & -3 \\ 10 & 12 & -6 \\ -20 & 4 & -2 \end{matrix}$$

19. Solve the following system by Gauss-Seidel

method: 28x+4y-z=32, x+3y+10z=24,

2x+17y+4z=35.

20. Find the inverse of the matrix by Gauss-

Jordon method:

$$A= \begin{matrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{matrix}$$

21.Find the dominant Eigen value and the corresponding Eigen vector

of the matrix

$$A= \begin{matrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{matrix}$$

22 Apply Gauss-Jordon method to find the solution of the system 10x+y+z=12,

2x+10y+z=13 , x+y+5z=7

23 Solve the boundary value problem $y'' = xy$; subject to the conditions

$y(0)+y'(0)=1, y(1) = 1$, taking $h = 1/3$ by finite difference method.

24.   Write down Laplace equation and its finite difference analogue and the

standard five-point formula.

25. Write down the finite difference scheme for solving the Poisson's equation

Section -C

1. Use Lagrange's formula to calculate f(3)
   from the following table.

   x : 0 1   2   4   5   6

   y : 1 14   15   5   6   19

2. Using Newton's divided Difference
   formula , find the value of f(8) and from
   the following table

   x : 4 5   7   10   11   13

   y : 48 100 294 900 1210 2028

3. Given x :   1   2   3   4   5   6 7   8

           y :   1   8   27 64 125 216 343 512

   Estimate f(7.5).Use Newton's formula.

4. Using Newton's divided difference
   interpolation , find the polynomial of the given
   data :

   x : -1   0   1   3

   y :   2   1 0 -1

5. Find the cubic line interpolation

   x: 1 2   3 4 5

   y: 1 0   1 0 1

6.Given the   x: 5   7   11   13   17
values

           f(x   392 1452 2366 5202
)

   Evaluate f(9) using Newton's divided
   difference formula.

7. If f(0) = 0 , f(1)=0, f(2) = -12, f(4) =0, f(5) = 600,

    f(7) = 7308 , find a polynomial that satisfies this data
    using Newton's divided difference interpolation formula.
    Find f(6).

8. Find the Lagrange's polynomial of degree 3 to fit
   the data y(0) = -12, y (1) = 0, y (3) =6 and
   y(4) =12. Hence find y(2).

9. Find a polynomial of degree two for the data by
    Newton's forward difference method

         x :   0   1   2 3   4   5   6   7
         y .   1   2   4 7   11 16 22 29

10   Find y (12) using Newton forward difference formula

given

                   x: 10   20 30 40 50
                   y: 46 66 81 93 101

11   Fit a natural cubic spline for the .following data:

                   x: 0   1   2   3
                   y: 1   4   0   -2

12. Derive Newton's divided difference formula.

13. The population of city in a census taken once in ten years is
    given below. Estimate the population in the year 1955.

             Year :   1951   1961   1971 1981
             Population in thousands : 35   42   58   84

14. Find f(x) as a polynomial in x from the given data:

                   x: 3   7   9   10
                   f(x): 168 120 72   63
                   rind find f(8).

15. Obtain the cubic spline for the following data:

x: - 1 0   1 2

y: -1   1   3 35

given that $y_0'' = y_3'' = 0$

17. Determine by Lagrange's interpolation method The percentage number of patients over 40 years using the following data:

Age over (x) years	30	35	45	55
number (y) of patients	148	96	68	34

18. Find a polynomial of degree 4, which takes the following values , using Newton's forward difference formula

x: 2   4   6   8   10

y: 0   0 . 1   0   0

19. Using Lagrange's interpolation formula , find y(10) from the following table :

x: 5   6   9   11

y: 12   13   14   16

20.From the following table of half— yearly premium for polices maturing at different ages, estimate the premium for polices maturing at age 46.

Age x :	45	50	55	60	65
Premium y:	114.84	96.16	83.32	74.48	68.48

21.From the divided difference table for the Following data:

x: -2   0   3   5   7   8

y: -792 108 -72 48 -144   -252

22.The following values of x and y are given:

x: 1   2   3   4

y: 1   .2   5   11

Find the cubic splines and evaluate y(1.5).

23.   Use Lagrange's formula to fit a polynomial to the data:

x: -1   0   2   3

Y: -8   3   1   12

and hence find y at x=1.

24. Find x when j=20 using Lagrange's interpolation formula for

the data:

x: 1   2   3   4

y: 1   8   27   64

24. Using cubic spline approximation, find $y(0.5)$ and $y'(1)$ given

$M_0 = M_2 = 0$ and

x: 0   1   2

y: 5   -4   3

25. The following data are taken fiom the steam table:

Temp $°C$:   140   150   160   170   180

Pressure kgfcm':   3.685   4.854   6.302   8.076   10.225

Find the pressure at temperature t=175 C, using Newton's

backward difference formula.

26. Fit a Lagrange's interpolating polynomial y= f(x)

and find f(5)

x: 1   3   4   6

y=(x): -3   0   30   13
                                2

27. Given the following data , find $y'(6)$ and the maximum value of y

X: 0   2   3   4   7   9

y :   4   26   58 . 112   466   922

28. Solve the boundary value problem y" = xy . subject to the conditions $y(0)+y'(0)=l$,

$y(1) = 1$, taking $h = 1/3$ by finite difference method.